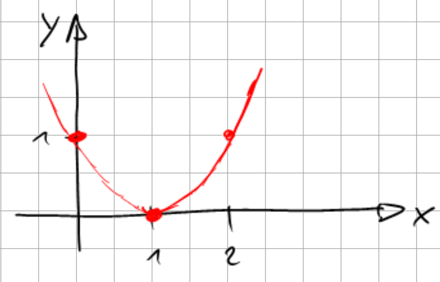
I Funktionen und Kurven

① $f(x) = x^2 - 2x + 1$
$= (x-1)^2$

x	0	1	2
y	1	0	1

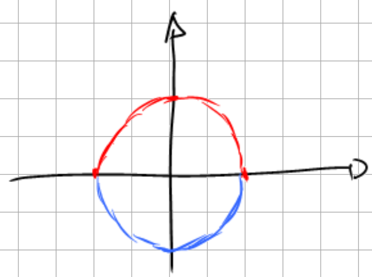


$f(x) = y = x^2 - 2x + 1$ (explizit)

$f: \quad x^2 + y^2 - 1 = 0$ (implizit)

$f: \quad y = \pm\sqrt{1-x^2}$ (explizit)

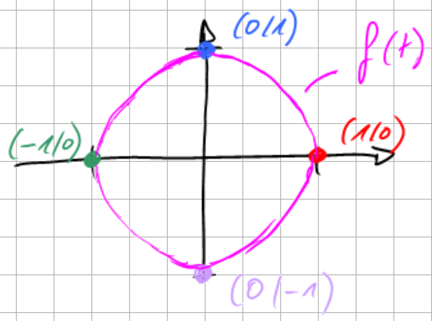
$-1 \leq x \leq 1$

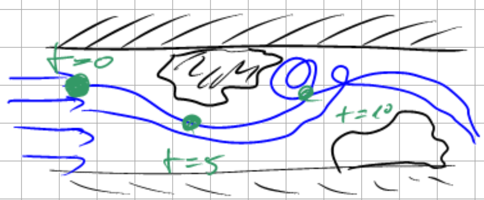


② $f(t) = (\cos(t) \mid \sin(t))$

Parameter — x-Koor — y-Koor

t	0	π/2	π	3π/2	2π
x(t)	1	0	−1	0	1
y(t)	0	1	0	−1	0





③ 2 Veränderliche

$f(x,y) = x^2 - y$

Veränderl.	x	0	1	2	0	1	2
	y	1	1	1	2	2	2
Ergebnis	z	−1	0	3	−2	−1	2

y \ x	0	1	2
1	−1	0	3
2	−2	−1	2

= z

1.2. Ganzrationale Funktionen / Polynomfunktionen

z.B. $f(x) = x^5 + 2x^4 - x^2 + 3x - 5$

$$f(x) = a_n x^n + a_{n-1} \cdot x^{n-1} + \dots + a_1 x + a_0 \qquad n \in \mathbb{N}$$

höchste Potenz von x $= n =$ Polynomgrad

1.2.1 Geraden

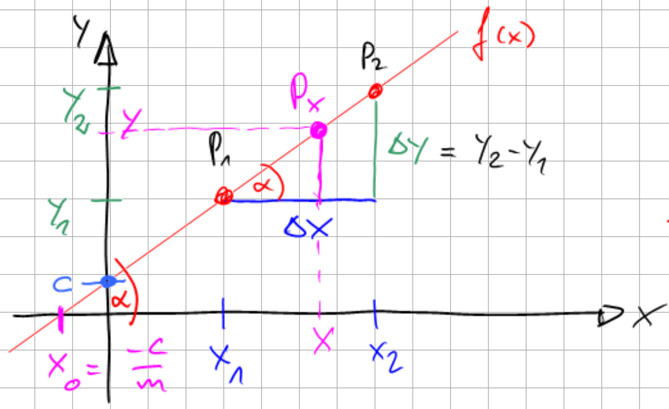
$f(x) = mx + c$ $=0$

$m = \frac{\Delta y}{\Delta x}$

c = Schnittpunkt mit y-Achse

$\tan\alpha = \frac{\Delta y}{\Delta x} \Rightarrow \alpha = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$

α = Steigungswinkel



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

Punkt-Steigungsform

2 Punkt-Form

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

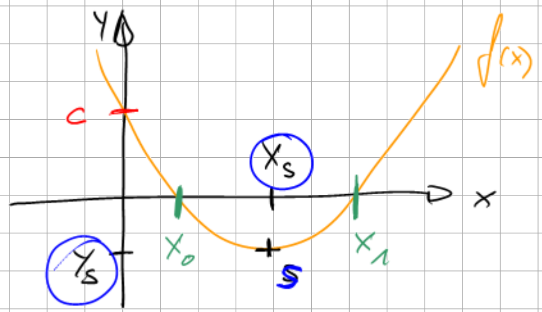
2 Punkte $P_1(x_1|y_1)$ $P_2(x_2|y_2)$

$$y = m(x - x_1) + y_1$$

1 Punkt $P_1(x_1|y_1)$

1 Steigung m

1.2.2. Parabeln



Hauptform:

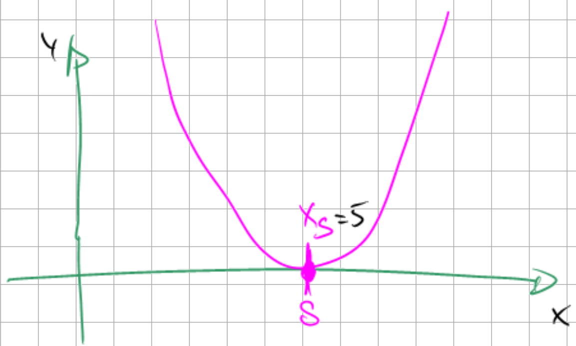
$$f(x) = ax^2 + bx + c$$

Scheitelform:

$$f(x) = a(x - x_s)^2 + y_s$$

Produktform:

$$f(x) = a(x - x_0)(x - x_1)$$

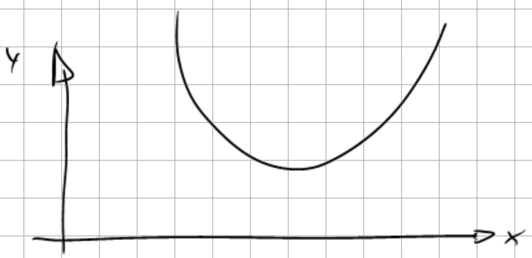


Bsp:

$$f(x) = a(x-5)^2 + 0 \quad \text{Scheitelform}$$

$$f(x) = a(x-5)(x-5) \quad \text{Produktform}$$

$$x_0 = x_1 = x_s$$



→ keine Nullstellen (NST) → keine Produktform

Bedeutung von a in den Funktionsgleichungen:

$a > 0$ → Parabel nach oben geöffnet 

$a < 0$ → —"— unten —"— 

$|a| = 1$ → Normalparabel 

$|a| < 1$ → flacher als Normalp. 

$|a| > 1$ → steiler als Normalp. 

Ermittlung von Nullstellen

Hauptform: $f(x) = ax^2 + bx + c$

allg. Lösungsformel: $x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

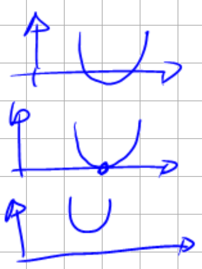
x_S

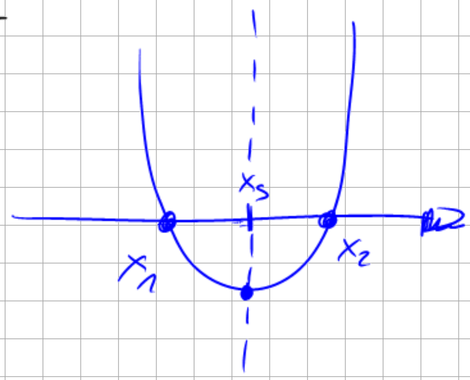
$\sqrt{D}$ $\quad D = b^2 - 4ac$

$D > 0 \Rightarrow$ 2 Lösungen

$D = 0 \Rightarrow$ 1 doppelte Lösung

$D < 0 \Rightarrow$ keine Lösung





Ermittlung der Scheitelkoordinate

$x_S = \frac{-b}{2a}$ $\quad y_S = f(x_S)$